

# Raymond Merrill Smullyan

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My initial plan for this talk was to discuss some simple examples of how Raymond's mathematics has been influential. I had two items in mind. *Uniform Notation* which has become common in automated theorem proving and some branches of proof theory, and *Elementary Formal Systems* which, unwittingly at the time, provided the semantics for Prolog. I had even begun preparing slides. But I realized this was not appropriate. The best discussion of Raymond's work is by Raymond himself. What is wanted here is a general discussion of the man and his remarkably many facets, and that is what I have decided to try for.

I knew Raymond longer than he knew me. I recently did some calculations and realized, much to my surprise, that I knew Raymond a bit short of  $\frac{3}{4}$  of my life. But I am just 75. He was 97 when he died, and had known me only a little more than  $\frac{1}{2}$  of his long life. I didn't meet him until he was in his mid 40's. Fortunately, Raymond wrote about his early life, others have written biographies, and I have drawn on them as well as on my personal memories.

Raymond Merrill Smullyan was born May 25, 1919 in Far Rockaway, a part of New York City very far from the city's centers. He said, more than once, how happy he was there. It is a curious fact that he was a classmate of Richard Feynman in grade school, though I never heard any inclination on Raymond's part to physics. (He did have an early interest in chemistry, however.) His father was a businessman from Belgium. The family was musical—his father played the violin and his mother the piano. Early on Raymond played both violin and piano, eventually concentrating on piano.

In 1929 the Great Depression hit, and Raymond's family had to move from Far Rockaway to the City, where he attended high school. This move saddened him deeply, and he mentioned it many times over the years.

Raymond came from a generally talented family. His brother Emile was an economist, and his cousin Robby became a painter and art dealer (specializing in commercial art, if I recall correctly). Raymond also had a sister, but I never met her and have almost no information. The extended family was very close, getting together frequently. Arthur and Robert Smullyan were cousins. Actually, they were 'double cousins': fathers were brothers and mothers were sisters. Arthur became a philosopher, best known for analyzing the problems with Quine's understanding of the scope distinction in Bertrand Russell's work on definite descriptions. Arthur first introduced Raymond to mathematical logic. Many years later, Raymond's brother Emile and his wife Etta bought a house in the Catskills, and Raymond found this so attractive that he and his wife Blanche built a house next door, where both of them would live out the rest of their lives.

Raymond was always someone who did things his own way. In high school he was undecided between a career in mathematics and one in music. He moved between high

schools, looking for one that was good in both areas, and eventually dropped out and studied on his own. I believe he got a GED instead of a conventional high school diploma. At that time he taught himself calculus, group theory, and Galois theory. And this is a place for a personal anecdote. He developed his own approach to Galois theory, perhaps at this time, perhaps later. I don't mean new theorems, but new ways of presentation and organization. Many years later when I was in graduate school I took his course in modern algebra, using his presentation of Galois theory. This would have been around 1965. I took quite detailed notes, which I still have. Perhaps two years ago he called me and asked about those notes. He wanted to turn them into a book. I sent them to him, but his health began to deteriorate and he felt his mind beginning to slow. He never wrote the book. but among the last things we spoke about was whether I would be willing to try writing it up. I told him I would try; but the outcome is uncertain. He owned the subject in a way that I do not. However this eventually comes out, it is a good example of Raymond's ability to focus on what interested him, over long periods, to the exclusion of whatever was felt to be unimportant in his life.

College was something like High School, in that Raymond came and went between several. While attending one, he met Bernhard Abramowitsch, who was a well-known pianist and teacher. (Perhaps Abramowitsch's best-known pupil was the composer David Del Tredici.) Raymond studied piano with Bernhard, and lived in his house in California for a time. This was the start of a long friendship. Many years later I met Bernhard at Raymond's house in the Catskills. I remember a short, friendly man with interests that certainly went beyond music. In the small-world way of things, Irene Schrier Scott recently wrote me: "You know, I have known Ray for most of my life, in so many different situations! Since I was nine years old, when he was studying piano with Bernhard Abramowitsch...in San Francisco. Much, much, much later he introduced me to Dana. Earlier, in Chicago, he often came to my parents' home, always with a whole train of the most interesting friends from the U. of C." The Dana mentioned is the mathematician and logician Dana Scott.

An aside. Raymond, despite his very public persona, was an intensely private person. He was very often "on" and would happily tell jokes and anecdotes, but getting to a real conversation took time and the right environment. Such conversations, when they happened, could be insightful and thought provoking, but personal details were often suppressed. For instance, I believe that somewhere around the time of college, he was married briefly. I never heard details, and am not entirely sure if it is true. This was typical. Things he did not want to talk about never got talked about. I remember, a number of years ago, Peter Abramowitsch, Bernhard's son, wanted to video an interview with Raymond as part of a projected documentary about him. He asked if I would take part, as a kind of interviewer. We set things up and I tried asking Raymond questions about events and feelings in the past, but they were all deflected with polite but uninformative answers. Eventually we simply gave up.

Raymond essentially dropped out of college, living with his parents in New York, without a job. It was during this time that he learned to perform magic, created chess puzzles, and continued studying modern algebra. Typical life of a dropout. He also

continued with his musical career. A biography he wrote mentions his playing piano in the Beethoven First Piano Concerto, which is interesting because, while his musical tastes were wider than one might have supposed, he eventually narrowed his own playing to Schubert, Scarlatti, Bach, and perhaps a few others. There are quite a few recordings of him playing these, many on YouTube, but I don't believe there is much else.

I mentioned chess puzzles as one of Raymond's interests during his dropout period. If you are not familiar with Raymond's puzzles, they involve retrograde analysis. Instead of asking what happens next, they ask what happened before. Typically a piece is missing and one wants to deduce information about it. This kind of puzzle was suggested to Raymond by his (double) cousin Arthur. Such problems are not really chess problems, but problems in logical deduction. Many years later Raymond built his puzzles into stories, and these were published as *The Chess Mysteries of Sherlock Holmes*, 1979 and *The Chess Mysteries of the Arabian Knights*, 1982. My first wife, Greer Fitting, did the illustrations for the Arabian Knights book, as she also did for his 1982 *Alice in Puzzleland*. I will not attempt to reproduce one of his chess puzzles here. Suffice it to say, he considered his best puzzle to be "Mystery of the Missing Piece" which he created when he was 16. It is in the Holmes book, and presentation of the reasoning going into the solution takes four pages.

Raymond attended several colleges, and finally received an undergraduate degree, which brings up a story Raymond was fond of telling. Without yet having a college degree, he received an offer from Dartmouth to be an instructor in mathematics, having been recommended by Carnap. He taught there for two years, and they wanted to keep him longer. The problem was, Raymond didn't have a college degree (or even a high school diploma). That's when the University of Chicago decided to give him credit for courses he hadn't taken, but had been teaching at Dartmouth, and that's how he managed to finally get his undergraduate degree from Chicago. Even so, he didn't remain teaching at Dartmouth, but became a graduate student at Princeton. He received a PhD with Alonzo Church as his mentor. This was in 1959, when he was already 40, a late start for an academic career. But his academic work actually begins earlier.

Raymond's first published paper preceded his degree and remained one of his most cited academic works, "Languages in which self reference is possible", appearing in the *Journal of Symbolic Logic* in 1957. In it he reduced Gödel's machinery for proving incompleteness to a minimum, establishing that such results held for formal systems substantially weaker than Peano arithmetic. In a sense he extracted the essential core of Gödel's argument, presenting it all in the elegant style for which he was to become known.

His 1959 dissertation gave rise to several published papers, and the dissertation itself was published in 1961 by Princeton University Press as Raymond's first book, *Theory of Formal Systems*. It introduced a remarkable range of influential material. In it his earlier work from "Languages in which self reference is possible" was developed as part of an extensive general approach. A new automata theoretic class called *rudimentary* was introduced. Fundamental results in recursion theory were proved, in particular a double recursion theorem. And what was of most interest to me personally, an extremely simple

formalization of recursion theory itself was created, *Elementary Formal Systems*. This reduced the machinery needed to define recursive enumerability and the recursive functions to an intuitively attractive minimum.

In the late 1960's there was much work concerning the notion of computation—in particular, what might computation mean over arbitrary structures. Many different approaches were introduced and equivalences were established. Much depended, for instance, on whether the structure admitted a recursive pairing function. But in two short abstracts in 1956 in the *Bulletin of the American Mathematical Society*, Raymond had already sketched elementary formal systems for arbitrary structures, and noted that on a structure of arithmetic it yielded ordinary recursion theory; on a structure of words over a finite alphabet it yielded Turing computability on words, and so on. Unfortunately this generality was not actually discussed in his book *Theory of Formal Systems*, and the extent of applicability was not generally known until years later.

For a number of years logic programming was an important topic of research in computer science. A semantics was developed for it (without negation as failure) by van Emden and Kowalski in 1976. It wasn't realized until later that Raymond's elementary formal systems, on the structure of formal terms, essentially coincided with Prolog, and the van Emden/Kowalski approach had already appeared in one of Raymond's 1956 abstracts. It should give some idea of the fruitfulness of Raymond's work, that important developments could lay there unrecognized because there was so much else to think about.

Raymond continued teaching as a lecturer, at Princeton, until 1961 at which point he moved to the Belfer Graduate School of Yeshiva University, where he remained until 1968. At this point my life began intersecting his. I went to an engineering college, Rensselaer Polytechnic Institute, graduating with a degree in mathematics in 1963. In college, one of the courses I took was based on Raymond's *Theory of Formal Systems*, which had just come out. I liked it very much. At the suggestion of the Professor who taught that course, I applied to Belfer Graduate School, where both Raymond and Martin Davis were. I was accepted, and started there in the Fall of 1963. I no longer remember dates and details, but I do remember that I took the same logic course with both Raymond and Martin (probably one was official, one not). It was enlightening to see just how different two approaches to the same material could be. Raymond made use of a preliminary version of what became his *First-Order Logic* book. Martin finished his course with a thorough presentation of work on Diophantine equations — Hilbert's 10<sup>th</sup> problem had not yet been solved. It was an exciting time.

I chose to work with Raymond. He was then, as he always was, an elegant and insightful teacher. Presentations were an art form. At that time Cohen forcing was new. Kripke modal semantics was new. I wound up combining the two. I remember telling Raymond that my dissertation was running long. He told me to write it as a book, which was excellent advice. It did become my first book.

Raymond's second book was *First-Order Logic*, in 1968. The idea, in part, was to simplify Beth's semantic tableau machinery and use it as the basis for theorem proving in classical first-order logic. Along the way the *Model Existence Theorem* was introduced

(with a different name), *uniform notation* was presented, a constructive proof of cut elimination was given in an abstract setting that yielded it simultaneously for both tableaux and the sequent calculus. This is not a complete list of innovations from that book. The book has influenced several generations of logicians, and has been reprinted in the Dover book series, in 2010. This work has also had an unanticipated life in computer science, in the field of automated theorem proving. The annual Tableaux conference has been meeting since 1992 and is devoted to the use of tableau methods applied to a wide range of logics. Most of the papers at these meetings trace back to Raymond's book in one way or another.

A side note. I remember when *First-Order Logic* was first printed, and Raymond was sent a copy. While the contents were fine, the cover announced *First-Order Logik*, with a Germanic "k". The printing had to be called back and redone. Probably there are no copies of this curiosity left now.

I received my PhD in 1968, and by this time Raymond was not just a mentor, but a friend. I came to know his wife Blanche, a dear and remarkable woman. Originally from Belgium, she was a cultured, independent person, who had founded a music school in Manhattan, though by the time I came to know her, the school was no longer being run. I mention a note from Irene Schrier Scott, "Did you know I had actually introduced him to Blanche? I had no idea what would ensue; but I am so happy it happened." It was a long and happy marriage, and Blanche was a good friend. Blanche passed away in 2006 at the age of 100, having been cared for at home by Raymond and assistants during the last year of her life.

In the 1960's Raymond and Blanche lived in the brownstone that had once housed Blanche's music school. Blanche was an excellent pianist, though she didn't seem to enjoy playing in public the way Raymond did. I visited there often, and stayed there from time to time when I had my farm upstate. The brownstone had rooms full of grand pianos, a Mozart piano, a clavichord. A remarkable place. Downstairs a small blackboard had been set up, and at one point Raymond explained Gentzen's proof of cut elimination to me, in terms of tableaux. I think his summary was: keep pushing cuts further down the tree, while reducing formula complexity. Eventually all cuts will be at branch ends and will be atomic, and these easily disappear. That simple summary, in fact, is quite enough to reconstruct the full proof. But let's skip over technicalities.

By 1968 Belfer Graduate School was having problems, and faculty were leaving. City University's Hunter College was splitting; the so-called uptown branch became a separate institution, Herbert Lehman College. Raymond and I both were hired there. It was a different time from today. The chair, Iz Rose, who later became not just a colleague but a friend, interviewed the two of us together, casually. I don't remember meeting a committee, or there being many formal complexities. I remained at Lehman, and the CUNY Graduate Center, for the next 44 years. Raymond stayed until 1982, when he became Oscar R. Ewing Professor of Philosophy at the University of Indiana, Bloomington, where he was until his retirement in 1989. I should mention that he hated the idea of retirement. He loved teaching. But at that time there was still a mandatory retirement age and nothing could be done about it. This is no longer the case, but the

change in the law came too late for Raymond. Nonetheless, retirement did not mean retirement in any concrete way, where Raymond was concerned.

Backing up a bit, and returning to the late 1960's, Raymond and Blanche decided to build a house in the Catskills, next door to Raymond's brother Emile and his wife Etta. I remember the house when it was just a pile of lumber. Unfortunately, things did not go well. The contractor was less than honest. The house stood higher than intended because the basement was not excavated to the proper depth. Windows leaked. A grand piano could not go upstairs because a support beam had been omitted from the downstairs ceiling. But over time other contractors corrected these things. One major step involved wrapping a cable around the house and pulling it tightly together, then constructing permanent ties inside to hold things after the cable was removed. Eventually it became a pleasant, livable house in a very beautiful part of the country. I once wrote a children's story, *George and the Sand Castle* (available on my web site). It's actually based on events at that house, though perhaps nobody but me would recognize this.

Visiting Tannersville (where the Smullyan house was) was always an adventure. There was a constant assortment of mathematicians, musicians, or what not. It was there I came to know Stanley Tennenbaum, for instance. Sometimes in the morning one would hear Raymond upstairs, playing Schubert. There were books on every subject. Local theater groups to visit. It was an isolated outpost that brought the big world to its living room.

In 1971 I was married for the first time, to Greer Woodruff. With my parents and my wife's sister attending, the marriage took place in the Smullyan living room. After a year or so my wife and I bought an abandoned farm, further upstate than Tannersville. Unlike the Smullyans, we were serious about farming. But that didn't rule out visits, and there were many. We had two children, both of whom have fond memories of visiting Raymond and Blanche.

At some point Raymond, having been attracted by a dog in a pet store window, came home with it, Peekaboo. Raymond was uncomfortable with having the dog neutered, so there were a couple of litters of pups before he changed his mind, reluctantly. The Smullyans kept (I think) five dogs altogether, and homes were found for the rest. We wound up with one, Benedict, a loyal and faithful friend through good times and bad.

My father-in-law, Oscar Collier, was a literary agent. He asked me if I would consider writing a book something like *Mathematics for the Millions*, which had sold well for years. I was not interested, but I knew Raymond had started developing logic puzzles with a popular appeal. Somehow I got the two of them together, and my father-in-law became Raymond's agent for many years.

Eventually our farm was sold, we were divorced in 1983, and I faced the job of raising two daughters. After some tumult, which we will skip over here, I wound up in Westchester, where I still live. My parents provided continuity for my daughters, of course, but so did Raymond and Blanche. This was a job for old friends, and these old friends did their job well.

I mentioned our dog Benedict. He had gone from the Catskills, to our farm, to my parent's house, to Westchester. Throughout he remained faithful, friendly, and a comfort to be around. Eventually he developed a sore on his leg and it would not heal. The vet recommended removing the leg and, after much thought, I agreed. In fact, the dog lived happily for many more years. But the procedure was very expensive, relative to my financial state at the time. I happened to mention this to Raymond once, and a few days later a check arrived covering the operation. Raymond was a kind and generous man, in his quiet way.

Time passed. It does so more quickly, it seems, as life goes on. I was married a second time in 1992, to Roma Simon, and we are happy together while aging less gracefully than we would like. My children became adults. Both now have children of their own. They visited Smullyans from time to time. Then Blanche died in 2006, at the age of 100. Visits continued, but distances made this difficult. Still, it was a remaining connection with their childhood. I am sorry my grandchildren will never know Raymond. They were very young when he was very old, and now it is impossible.

The rush of mathematical creativity that Raymond experienced in the late 1950's gradually evolved to a more encompassing point of view. Raymond wrote several books giving his mature thoughts about the topics that had always occupied him since his dissertation, *Theory of Formal Systems*. These books are *Gödel's Incompleteness Theorems*, 1992, *Recursion Theory for Metamathematics*, 1994, and *Diagonalization and Self-Reference*, also 1994.

While I was at Belfer Graduate School, Raymond lectured on set theory. He had worked out very nice treatments of ordinals, the axiom of choice and its relatives, constructible sets, and class-set theory. He did not really grasp the new method of forcing. My dissertation had been on forcing. One day in the 1990's we were visiting Tannersville and Raymond suggested the two of us write a book on set theory. And we did. The first two parts were based on Raymond's class notes. The third part on forcing was a redevelopment on my part of my thesis work, bringing modal logic into it in a way that I thought was more natural than my original approach. The resulting book *Set Theory and the Continuum Problem* appeared in 1996, and is now in a second edition, with Dover. Surprisingly, this is our only joint work. In fact, Raymond has almost no joint work. There is a joint paper with Anil Nerode in 1962, and one with Hillary Putnam in 1960, but that's it.

As I mentioned before, at some point in the 1970's, while continuing his mathematical logic researches, Raymond developed an interest in puzzles that are based on logic. Martin Gardner devoted a column in the magazine *Scientific American* to some of these, and they turned out to be quite popular. This led to *What is the Name of This Book*, 1978. It was quite successful, and was followed by other books: *This Book Needs No Title*, 1980, *Alice in Puzzle-Land*, 1982, and many, many more. There have been a large number of translations into many languages, and non-academics probably know Raymond entirely through these works.

Raymond realized that puzzles could be used to get across some of the fundamental discoveries of modern logic. This led to his book *The Lady or the Tiger?* in 1983, which

took readers through the basic ideas of Gödel's incompleteness theorem, via a series of puzzles. This was followed by *To Mock a Mockingbird*, 1985, which explored the Lambda calculus via puzzles about birds. The Lambda calculus is a system of formal logic with applications to the design and semantics of computer languages. Raymond's book was of such interest that the automated theorem-proving group at Argonne National Laboratories once developed a program specifically for solving his so-called bird puzzles. Another work along these lines is *Forever Undecided*, 1987, which discusses Gödel's theorem in greater detail than before, and also modal logic and provability logics. Raymond's book, *Satan, Cantor, and Infinity*, 1992, discusses issues of probability, infinity, time, and change, largely through his familiar medium of puzzles. And I should mention *The Magic Garden of George B and Other Logic Puzzles*, 2007, which uses puzzles to present Boolean algebra and the Stone representation theorem.

In his last works Raymond turned back to a more conventional format and wrote two textbooks which he hoped would be useful in the teaching of first-order logic and its metatheory. These are *A Beginner's Guide to Mathematical Logic*, 2014, and *A Beginner's Further Guide to Mathematical Logic*, his last book, published in 2016.

Raymond's bibliography is available on the web site for this conference. I did some counting. Raymond published 8 mathematical books, and 22 popular books, including the puzzle books. There was also one book he co-edited, about pianists at The Piano Society. Not bad for a dropout.

Raymond had numerous interests outside logic. He was intense, concentrating on these sometimes to the exclusion of all else, but his personal interests were, in fact, of interest to many.

As we heard earlier, Raymond was a first-rate pianist and once considered a career as such. Essentially, a bout of tendonitis put an end to this as a career. Several of his performances can be found on YouTube, and one CD is available for purchase.

[Video: [RaymondPiano2000 \(2:51\)](#)]

In his younger days Raymond supported himself as a professional magician, doing slight of hand at tables in a nightclub. Almost to the end of his life, when he visited a restaurant he would bring a pack of cards and would go from table to table entertaining diners. I've seen this happen often, and always a good time was had by all.

[Video: [Romania2008 \(4:16\)](#)]

There were lesser interests that would have been major for anyone else. He had a lifelong enthusiasm for astronomy and when I first knew him, he was making his own (quite large) telescopes, grinding his own lenses. Much later he became interested in sound reproduction, and put together some very formidable speaker systems with somewhere around 40 speakers, each of which could be individually controlled, to experiment with sound balance and related aspects of music reproduction. Raymond also experimented with three-dimensional photography for many years. He developed a way to build viewers that were better and cheaper than those commercially available, and even published an article about it.

At some point Raymond became interested in making videos. This required the use of a computer, and so he purchased one, an Apple Mac. I regularly got calls asking how to do something or other. These ranged from “where is the semi-colon key” to how to use iMovie to edit video. He became quite good at it. The culmination was a video he made in 2006, *Remembering Blanche*, a tribute to his wife who had recently died. It was a remarkable thing. He combined still photos with videos others had taken over many years, and ended with a moving series of pictures of Blanche taken the day before she died. It was a direct look at the cumulative effects of time, seen in face of someone he loved and who no longer related to the world around her.

Although Raymond claimed to not know if he was religious, he had a deep interest in eastern religions. Probably his best-known writing in this area is *The Tao is Silent* from 1977.

[Video: [ColdMountain2016 \(2:32\)](#)]

I last visited Raymond some time in January. He had become very frail. A full time attendant was necessary. Raymond could not even move from one chair to another by himself. Still, twice a day he and his attendant would go out to eat. Perhaps the human contact was necessary to him. His conversation remained lively. It was about the political situation, philosophy, all the old things, except for much about mathematics. He felt his mind was no longer quick enough to follow mathematical arguments. In general he was still interested, and interesting. But he was waiting for it to be over. On my previous visit he had turned to me at one point and asked, “Doesn’t it make you sad to see me this way?” He smiled as he said it, but it did make me sad, and I said so. His waiting is now over. Raymond passed away February 6, 2017, at the age of 97. To quote Irene Schrier Scott one more time, “Somehow, Raymond was supposed to go on living forever - just in some magical way, be transformed into pure spirit.”

Talk given at *Remembering Raymond Smullyan*, CUNY Graduate Center, April 27, 2016.